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Optimization of Project Duration in Uncertain Condition through Fuzzy Logic Approach

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ABSTRACT

Construction industry is very complex system and needs to adapt most systematic as well as rational approach to optimize project duration to complete undertaken projects within stipulated time, budget and required quality. In process of optimization, a large number of interactive variables of the project creates problem, so need to determined there impact to reach any decision. Linear programming model help to optimize decision making in rigid environment, but in real working environment where goals, constrains, consequence of action are not known precisely, uncertainty play major factors toward project success and performance, need certain degree of flexibility to be incorporated in decision making model to optimize project duration. This paper demonstrates with fuzzy linear programming model to incorporate flexibility in project scheduling with a case study.

Keywords : Fuzzy set, Linear Programming model, Fuzzy linear Programming, Optimization in Uncertain Environment.

I. INTRODUCTION

Most challenging jobs that any construction manager can take on are the management of a large scale project that requires coordinating numerous activities throughout project construction phase. A myriad of detail must be considered in planning how to coordinate all these activities, in developing a realistic schedule and then monitoring the progress of project. Organizations worldwide continue to have issues with completion of project on time, on budget, with high client satisfaction. This issue has been documented in the Construction industries. Research has shown the overall performance of services is poor with 2.5% of projects are defined as successful (scope, cost, schedule, and business), only 30% of projects are completed within planned cost and schedule, 25 to 50% is wasted due to problem of coordinating labor on a project, and management inefficiency.

Project planning is important aspect in construction Industry to complete the project within stipulated time and budget with required quality and safety. Any construction project can become reality only when it is technologically and financially feasible. Highly variable and unpredictable factors influence construction process in greater extent. The effective management techniques are necessary to control time- resources- cost and achive success of project in reality. Application of Operation Research (OR) techniques to construction project problem is quite common to achieve goals.

Operation Research involves construction of mathematical model of decision and control problem to treat with complexity and uncertainty within system. When all factors affecting the system are known mathematical model can be developed and linear programming is often used for allocating scares resources with objective of making optimal uses of them.

As in linear programming both objectives function and constraints are rigid but practically they are variable such as situations, need to confront with scares resources and at time when project expected to complete at earliest due to additional availability of resources, same time constraints of completion of one activity to start another activity can't expected accurately due to delays or early start time of preceding or succeeding activity. So the problem of optimal allocation of resources in efficient manner to minimize duration and to obtain maximum profit is great challenge to construction industry. To incorporate a certain degree of flexibility in order to get realistic duration of project with fuzzy set concept, this paper applied fuzzy linear programming to identify optimal construction project completion duration. A case study of model construction project is also considered in this course to estimate optimize project duration.

II. METHODS AND MATERIAL

A. Fuzzy Set Theory

One can view fuzzy set as generalization of classical set or crisp set. Generally crisp set defined as a collection of objects that may share certain characteristics, so elements of crisp set are precise and sharp, unambiguous distinction exits between members and nonmembers. One may define set of positive integer $\mathbb{N} = \{1,2,3,\ldots\}$ i.e. element x is either a member of given set $\mathbb{N}$ ($x \epsilon \mathbb{N}$) or not a member ($x \notin \mathbb{N}$), partial membership not allowed. A set is defined by a function, called Characteristic Function that declares which elements are members of set and which are not. Set A defined by characteristic function, χ_A as follows:

$$\chi_A(x) = \begin{cases} 1 & for\ x \in A \\ 0 & for\ x \notin A \end{cases}$$

i.e. characteristic function maps universal set X to elements of set {0,1} ; $\chi_A : X \to \{0,1\}$. This function can be generalized such that values assigned to the elements of universal set fall within a specified range and indicate membership grade of these elements in set in questions. Such function is called membership function and set defined by it a fuzzy set. Membership function of a fuzzy set $\tilde{A}$ maps elements of given universal set X, which is always a crisp set into real numbers in [0,1] i.e. $\mu_{\tilde{A}} : X \to [0,1]$

Thus fuzzy set is a set where degrees of membership between 1 and 0 are allowed i.e. it allowed partial membership and can better reflect the way intelligent people think. As an intelligent people not classify people as either friends or enemies there is a range between these two extreme. Fuzzy set theory developed specifically to deal with uncertainties that are not statistical in nature and use to model imprecision, ambiguity or fuzziness in formulation. Membership function allows various degrees of membership for the elements in X of a given fuzzy set $\tilde{A}$, is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}(x)}) | x \in X\}$

$\mu_{\tilde{A}(x)}$ = membership function / grade of membership / degree of truth of x in $\widetilde{A}$.

Let X= universal set; x= is set of element and A = subset of X; Characteristic function, for fuzzy set

$$\mu_A = \begin{cases} \in [1,0]\ if - a \le x \le a;\ x \in A \\ 0 \quad otherwise\ ;\ x \notin A \end{cases}$$

and for crisp set (illustrate in Figure 1)

$$\chi_A(x) = \begin{cases} 1 \quad for\ - a \le x \le a;\ x \in A \\ \quad 0 \quad for\ x \notin A \end{cases}$$

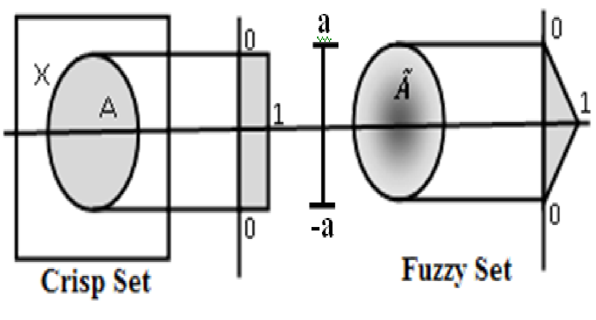


Figure 1. Difference between Crisp set and Fuzzy set.

B. Optimization in Uncertain Environment or Fuzzy Decision Making

The subject of Decision making is the study of how decisions are actually made and how they can be made better and more successfully. Field is concerned, in general, with both descriptive theories, normative theories. Much of the focus in developing the field has been in area of management, in which decision making process is key importance for functions such as Allocation of resource, inventory control, investment, personnel actions, as well as many others. Decision making is very important part in project planning, implementation in construction Industry. In this course they deal with uncertain situations due to different uncertain variables present in project environment.

Applications of fuzzy sets within the field of decision making have, for the most part, consisted of fuzzification of the classical theories of decision making [1]. In classical (normative, statistical) decision theory, a decision can be characterized by a set of decision alternatives (the decision space), a set of states of nature (the state space), a relation assigning to each pair of a decision and state a result and finally, the utility function that orders the results according to their desirability.

When deciding under certainty, the decision maker knows which state/ outcomes to expect and chooses the decision alternative with the highest utility, given the prevailing state of nature. When deciding under risk, i.e. does not know exactly which state will occur, only knows a conditional probability distribution one for each alternatives action. When probabilities of outcomes are not knows or may not even be relevant and outcomes for each alternatives action are characterized only approximately, decision are made under uncertainty is prime domain of fuzzy decision making.

Fuzziness can be introduced into existing model in different ways. Bellman and Zadeh [2] suggest fuzzy model of decision making in which relevant goals and constraints are expressed in terms of fuzzy sets, decision is determined by an appropriate aggregation of these fuzzy sets.

The objective function as well as constrains is fuzzy in project environment and relationship between constraints and objective function in fuzzy environment is symmetric; decision is confluence of goal and constraints [3].

Assume that a fuzzy goal $\tilde{G}$ and a fuzzy constraint $\tilde{C}$ in a space of alternatives X is known. Then $\tilde{G}$ and $\tilde{C}$ combine to form a decision $\tilde{D}$, which is a fuzzy set resulting from intersection of $\tilde{G}$ and $\tilde{C}$. In symbols, $\tilde{D} = \tilde{G} \cap \tilde{C}$, and correspondingly, $\mu_{\tilde{D}} = \min(\mu_{\tilde{G}}, \mu_{\tilde{C}})$

$\mu_{\tilde{D}} =$ membership function of fuzzy set decision, $\mu_{\tilde{G}} =$ membership function of fuzzy set goal

$\mu_{\tilde{C}} =$ membership function of fuzzy set constraints

More generally, suppose that n goals $\tilde{G}_1, \tilde{G}_2, \dots, \tilde{G}_n$ and m constrains $\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_m$, then resultant decision is intersection of given goals and constrains i.e. $\tilde{D} = \tilde{G}_1 \cap \tilde{G}_2 \cap \dots \cap \tilde{G}_n \cap \tilde{C}_1 \cap \tilde{C}_2 \cap \dots \cap \tilde{C}_n$ and correspondingly $\mu_{\tilde{D}} = \min\{\mu_{\tilde{G}_1}, \mu_{\tilde{G}_2}, \dots\dots, \mu_{\tilde{G}_n}, \mu_{\tilde{C}_1}, \mu_{\tilde{C}_2}, \dots\dots, \mu_{\tilde{C}_m}\} = \min\left\{\mu_{\tilde{G}_i}, \mu_{\tilde{C}_j}\right\} = \min\{\mu_i\}$

So there are three assumptions:

a> The "and" connecting Goals, constrains in model corresponds to "logical and".

b> The "Logical and" corresponding to set theoretic intersection.

c> The intersection of fuzzy sets is defined by min operator.

In short, a broad definition of the concept of decision may be stated as: Decision = Confluence of Goals in Constraints.

As the fuzzy set "decision" is characterized by membership function $\mu_{\tilde{D}} = \min(\mu_{\tilde{G}}, \mu_{\tilde{C}})$ i.e minimum of $\mu_{\tilde{G}},\mu_{\tilde{C}}$ illustrate in Figure 2. If the decision maker wants to have a crisp decision proposal, the highest degree of membership in the fuzzy set decision is suggested [4,5]. This is call "maximizing decision", defined by

$a^* = \arg(max \min(\mu_{\tilde{G}},\mu_{\tilde{C}}))$

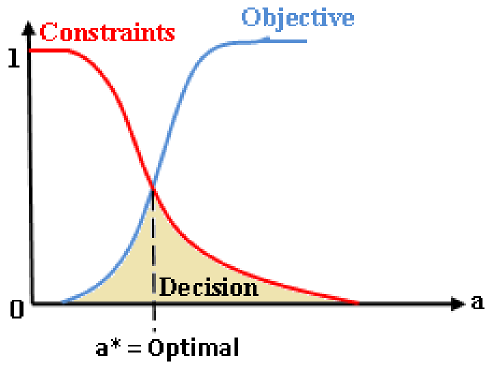


Figure 2. Maximizing Decision

C. Linear Programming

Linear programming (LP) deals with the problem of allocating available limited resources among competing activities in an optimal manner. Classical LP problem is to find minimum or maximum values of a linear function under constraints represented by linear inequalities or equations.

Minimum or Maximize $C_1x_1 + C_2x_2 + \cdots . + C_nx_n$

Subject to $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$

…………..

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$

$x_1, x_2, x_3, \dots, x_n \geq 0$

Where, x_i= Decision Variables C_i= Cost Coefficients a_{ij}= Technological Coefficient b_i= Resource Values

In convenient matrix notation a typical LP can be express as-

Max /Min Z= C^Tx

Subject to, $Ax \leq b \; and \; x \geq 0$

where, Z= Objective function; T= Transpose Matrix . $C^T = (C_1, C_2, \dots, C_n)^T$ Vector of known constant containing coefficients in objective function $x = (x_1, x_2, \dots, x_n)$ vector of decision variables $A[a_{ij}] i \in \mathbb{N}_m \, j \in \mathbb{N}_n$ is matrix containing technological coefficient i.e. constraint matrix b= $(b_1, b_2, \dots, b_m)^T$ is RHS values of constraints. Set of x that satisfies all given constraints is called feasible set.

In practical situations it is not reasonable to require that constraints or the objective function in linear programming problem be specified in precise, crisp terms. Since real world problems does not go with rigidity, need to introduce flexibility in LP model. Such situation it is desirable to use fuzzy linear programming.

D. Fuzzy Linear Programming:

Fuzzy linear programming is extremely used for decision making in uncertain environment. Taking assumption that LP-decision has to be made in fuzzy environments, quite a number of possible modifications of LP model depends on what and where fuzziness is to be introduced, which may be initiated in following ways:

Firstly, decision maker might not really want to actually maximize or minimize the objective function. Rather, want to reach some aspiration levels that might not even be definable crisply.

a> The fuzzy Goal i.e. Maximum of linear Objective function is expressed vaguely and usually with aspiration level and it has flexibility e.g. The target value of objective function C^Tx is maximum as possible and pursued an aspiration level.

Secondly, the constraints might be vague in one of the following ways : ≤ sign might not be meant in the strictly mathematical sense, but smaller violations might well be acceptable. This can happen if the constraints represent aspiration levels, for instance, the constraints represent sensory requirements (taste, color, smell, etc.) that cannot adequately be approximated by a crisp constraint. Of course, the coefficients of the vectors b or C or of the matrix A itself can have a fuzzy character either because they are fuzzy in nature or because perception of them is fuzzy.

a> The Fuzzy Constrains i.e linear system of constrains expressed by fuzzy relations in terms of fuzzy equations or/and fuzzy inequalities.

b> The Objective function with fuzzy cost coefficient $\widetilde{C_i}$

c> The linear system of constrains with fuzzy technical coefficient $\widetilde{A_{ij}}$ and/ or fuzzy resources $\widetilde{B_i}$

Finally, the role of the constraints can be different from that in classical linear programming, where the violation of any single constraint by any amount renders the solution infeasible. The decision maker might accept small violations of constraints but might also attach different degrees of importance to violations of different constraints.

a> Constrains can be represented by fuzzy set rather than crisp inequalities.

Fuzzy linear programming offers a number of ways to allow for all these types of vagueness, and one way to be considered as Bellman-Zadeh's concept of a symmetrical decision model.

Consider a model problem to maximize an objective function subject to constraints.

Maximize $Z = f(x) = C^T x$

Subject to $Ax \leq b, x \geq 0$ ….(1)

A, b,C describe relevant state variables; x decision variables; Z resulting from combination of state and decision variables. Objective function is expressed by requirement to maximize Z.

Now introducing subjective aspiration level for value of the objective function (Z) want to achieve, and fuzzifying the equations in appropriate linguistic interpretations, each constraints is modeled as fuzzy set, LP is transformed into a fuzzified version:

Find x such that $C^T x \gtrsim Z$; $Ax \lesssim b; x \geq 0$ … (2)

H $ere \lesssim$ denote fuzzified version of $\leq$ has the linguistic interpretation "essentially smaller than or equal to" and $\gtrsim$ denote fuzzified version of $\geq$ has the linguistic interpretation "essentially greater than or equal to". Objective function is minimizing goal in order to consider Z as an upper bound i.e. $-C^T x \lesssim -Z$ model (2) is fully symmetric with respect to objective function and constraints. Substituting $\binom{-C}{A} = B$ and $\binom{-Z}{b} = d$

Find x such that $Bx \lesssim d; x \geq 0$ … (3)

Each of the *(m* + 1) rows of model (3) shall now be represented by a fuzzy set, the membership functions of which are $\mu_i(x)$. So $\mu_i(x)$ can be interpreted as the degree to which x fulfills/ satisfies fuzzy inequality $B_i x \leq d_i$ here B_i denote ith row of B and right hand side number d_i are fuzzy number. Therefore, the membership function of the fuzzy set "decision" of model (3) is

$\mu_{\widetilde{D}}(x) = \min_i\{\mu_i(x)\}\ \forall\ i = 1,2,3, \dots, m+1$ …(4)

Assuming that the decision maker is interested not in a fuzzy set but in a crisp "optimal" solution, then it could suggest

$$\max_{x\geq 0} \min_i\{\mu_i(x)\} = \max_{x\geq 0} \mu_{\widetilde{D}}(x) \text{ ... (5)}$$

Now specify the membership functions $\mu_i(x)$ should be 0 if the constraints (including the objective function) are strongly violated, and 1 if they are very well satisfied (i.e., satisfied in the crisp sense); and $\mu_i(x)$ should increase monotonously from 0 to 1, that is (ref to Figure 3).

$$\mu_i(x) = \begin{cases} 1 & if B_i x \leq d_i \\ \epsilon[1,0] & if\ d_i < B_i x \leq d_i + p_i \\ 0 & if\ B_i x > d_i + p_i \end{cases} \quad \forall i = 1,2, \dots, m+1 \text{ …(6)}$$

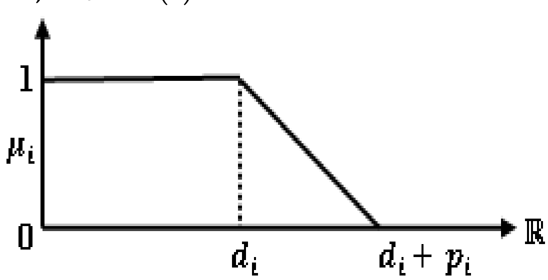


Figure 3. Fuzzy number.

Using the simplest type of membership function, assuming them to be linearly increasing over the "tolerance interval" p_i , :

$$\mu_i(x) = \begin{cases} 1 & if B_i x \le d_i \\ 1 - \frac{B_i x - d_i}{p_i} & if\ d_i < B_i x \le d_i + p_i \\ 0 & if\ B_i x > d_i + p_i \end{cases} \quad \forall i = 1,2, \dots, m+1 \quad \dots(7)$$

The p_i are subjectively chosen constants of admissible violations of the constraints and the objective function. Substituting equation (7) into model (5) yields, after some rearrangements [Zimmermann 1976] and with some additional assumptions,

$$\max_{x \ge 0} \min_i \left\{ 1 - \frac{B_i x - d_i}{p_i} \right\} \dots (8)$$

Introducing new variable λ and flexibility p_i get max λ

such that $\lambda p_i + B_i x \le d_i + p_i \quad \forall i = 1, \dots, m+1$

$0 \le \lambda \le 1; x \ge 0 \dots(9)$

The optimal solution to problem (9) is the vector (λ, x_0) then x_0 is the maximizing solution of (5) of model (2), assuming membership functions as specified in (7).

So far, the objective function and all constraints were considered fuzzy. If some of the constraints are crisp, $Dx \le b'$, then these constraints can easily be added to formulations.

max λ

such that $\lambda p_i + B_i x \le d_i + p_i \ \forall\ i = 1, \dots, m+1$

$$Dx \le b'$$

$0 \le \lambda \le 1; x \ge 0 \dots(10)$

Let us now turn to the case in which the objective function is crisp and determination of a crisp "maximizing decision" by aggregating the objective function after appropriate transformations with the constraints. By considering the objective function to be crisp and by adding a set of crisp constraints $Dx \le b'$:

By maximizing

$f(x) = C^T x$

such that $Ax \lesssim b; Dx \le b'; x \ge 0 \dots (11)$

Let the membership functions of the fuzzy sets representing the fuzzy constraints be defined in analogy to equation (7).

$$\mu_i(x) = \begin{cases} 1 & if A_i x \le b_i \\ \dfrac{b_i + p_i - A_i x}{p_i} & if\ b_i < A_i x \le b_i + p_i \\ 0 & if\ A_i x > b_i + p_i \end{cases}$$

$\forall i = 1,2, \dots, m+1$

The membership function of the objective function (5) can be determined by solving the following two LPs:

Maximize $f(x) = C^T x$

such that $Ax \le b; Dx \le b'; x \ge 0 \quad \dots.(12)$

yeilding $f_1 = (C^T x)_{opt}$

and

Maximize $f(x) = C^T x$

such that $Ax \le b + p; Dx \le b'; x \ge 0 \quad \dots(13)$

yeilding $f_0 = (C^T x)_{opt}$

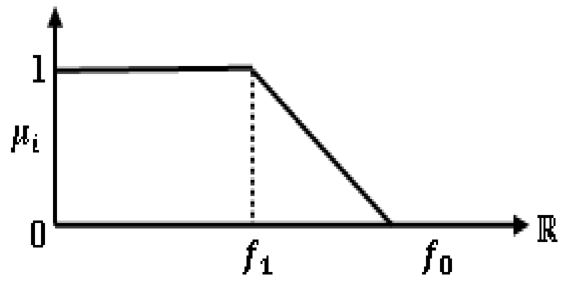


Figure 4. Fuzzy constraint.

The membership function of the objective function is therefore:

$$\mu_{\tilde{G}(x)} = \begin{cases} 1\ if\ f_0 \le C^T x \\ \dfrac{C^T x - f_1}{f_0 - f_1}\ if\ f_1 < C^T x < f_0 \\ 0\ if\ C^T x \le f_1 \end{cases}$$

This achieved "symmetry" between constraints and the objective function, and can employ the approach used to derive model (9) as an equivalent formulation of model (2). The equivalent model to (6) is:

$Maximise\ \lambda$

Such That $\lambda(f_0 - f_1) - C^T x \le -f_1$

$$\lambda p + Ax \le b + p$$

$Dx \le b';\ 0 \le \lambda \le 1; x \ge 0 \quad \dots..(14)$

Flexibility introduce in various elements of Fuzzy linear programming model using trapezoidal representation as figure 4 Methodology has been illustrated by a case study.

III. RESULTS AND DISCUSSION

A. Case Study

The activities and their duration involving into construction of a typical building project are listed in table 1.

Table 1. List of Activities.

Activity	Activity Description	Immediate predecessor	Estimated duration in days
A	Equipping the site and dismantling	--	14
B	Earthwork excavation for foundation	A	7
C	Concreting the footer	B	21
D	Erecting the frames for column and concreting	C	14
E	Slabs and beams	D	21
F	Brickwork	E	21
G	Laying sewage pipes	F	7
H	Filling the basement	G	7
I	Rough plumbing	F	7
J	Electrical wiring	F	7
K	Provision for air conditioning	H,I	14
L	Plastering	K,J	21
M	Flooring	L	21
N	Acoustic arrangement	L	7
O	Finished plumbing	N	7
P	Fixing doors and windows	M	7
Q	White washing and finishing	P,O	21

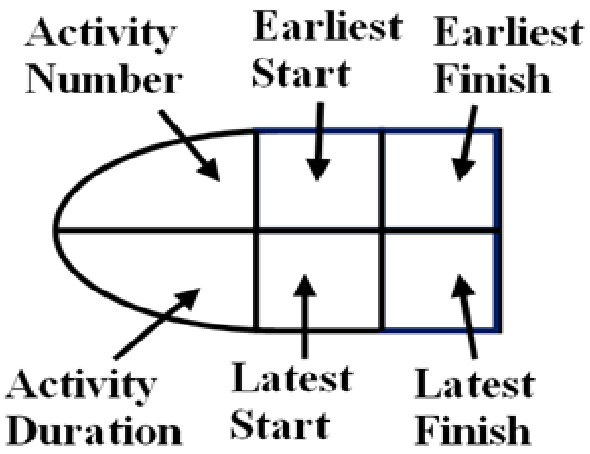


Figure 5. Activity-on-node configuration.

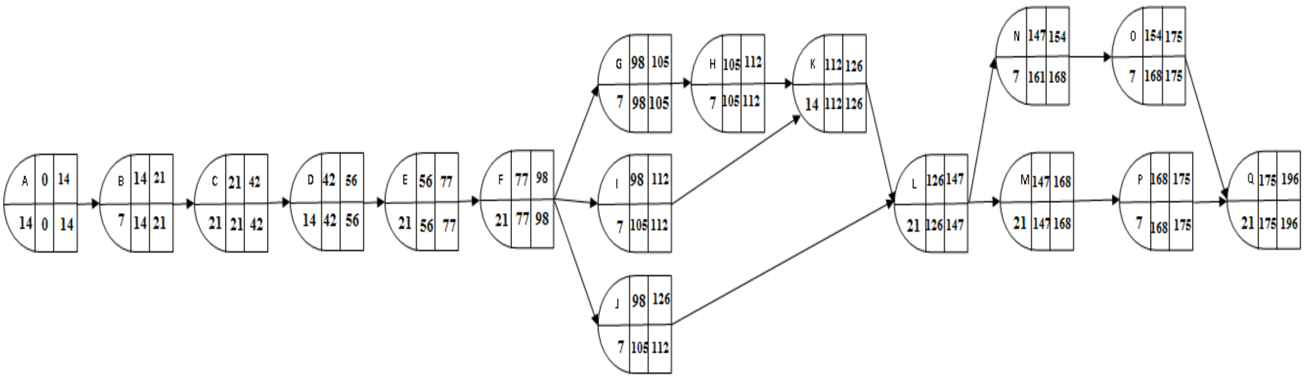
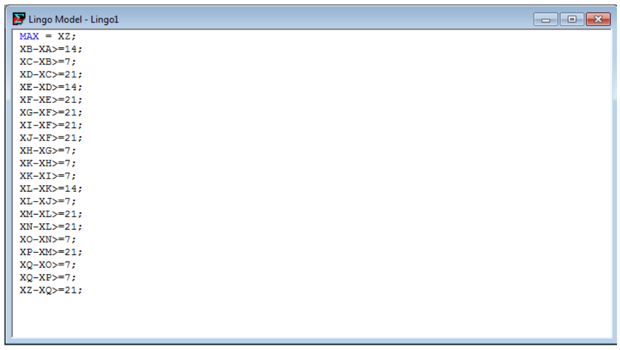


Figure 6. CPM network Diagram.

B. Linear Programming Formulation

The objective function for the linear programming model is to minimize project duration. $X_i (\forall i = A, B, C, \ldots)$ represent activities in network i.e. X_A represent Equipping the site and dismantling with duration 14, X_B represent Earthwork excavation for foundation with duration 7 then difference between earliest event times of X_B and earliest event times of X_A at least as great as the activity time duration of activity X_A. A set of constraints that expresses this condition is defined as: $X_B - X_A \geq 14$. Similarly other constraint equations are formulated using network.

The objective of the project network is to determine the earliest time the project can be completed (i.e., the critical path time). So the earliest event time of the last node in the network equals the critical path time. Implicitly assuming $X_A, X_{B,\ldots,} X_Q, X_Z \geq 0$; X_Z is time to completion of project, solution of linear programming model through LINGO (Figure 7) result Project Duration of 196 days i.e. equation (12). Yielding $f_1 = (C^T x)_{opt}$=196 days.



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Lingo Model - Lingo1
MAX = XZ;
XB-XA>=14;
XC-XB>=7;
XD-XC>=21;
XE-XD>=14;
XF-XE>=21;
XG-XF>=21;
XI-XF>=21;
XJ-XF>=21;
XH-XG>=7;
XK-XH>=7;
XK-XI>=7;
XL-XK>=14;
XL-XJ>=7;
XM-XL>=21;
XN-XL>=21;
XO-XN>=7;
XP-XM>=21;
XQ-XO>=7;
XQ-XP>=7;
XZ-XQ>=21;
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Figure 7. analysis for LP model using LINGO.

C. Fuzzy Linear Programming:

Due to presence of real world uncertainties like availability of resource, or delay in activity completion or others, solution of above linear optimization does not reveal true picture of project duration.

Flexibility is incorporated in constraints equations using fuzzy linear programming and above LP model is transform to equation (14) with different aspiration level introduced into duration of activities.

Table 2. List of activities with aspiration level duration.

Activity	Activity Description	Immediate predecessor	Original Estimated duration (b_i) in days	Aspiration level duration ($b_i + p_i$) in days
A	Equipping the site and dismantling		14	14
B	Earthwork excavation for foundation	A	7	7
C	Concreting the footer	B	21	21
D	Erecting the frames for column and concreting	C	14	18
E	Slabs and beams	D	21	21
F	Brickwork	E	21	16
G	Laying sewage pipes	F	7	7
H	Filling the basement	G	7	7
I	Rough plumbing	F	7	7
J	Electrical wiring	F	7	7
K	Provision for air conditioning	H,I	14	9
L	Plastering	K,J	21	17
M	Flooring	L	21	23
N	Acoustic arrangement	L	7	7
O	Finished plumbing	N	7	7
P	Fixing doors and windows	M	7	7
Q	White washing and finishing	P,O	21	21

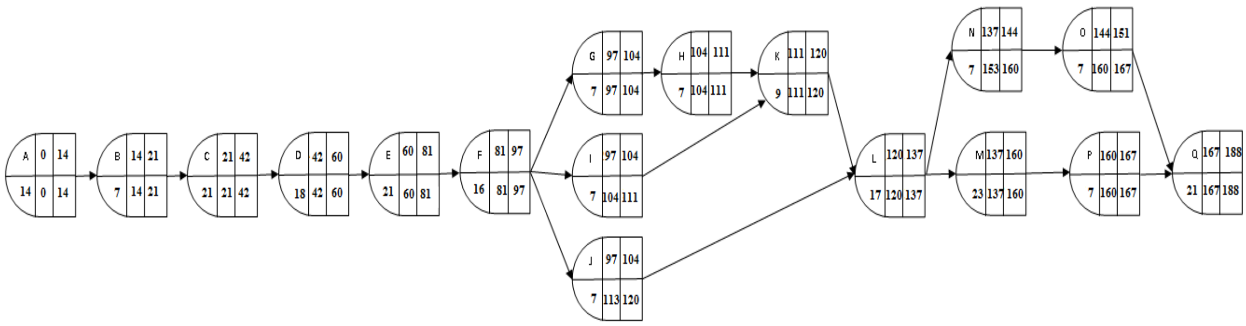

Figure 8. CPM network Diagram with aspiration level of duration.

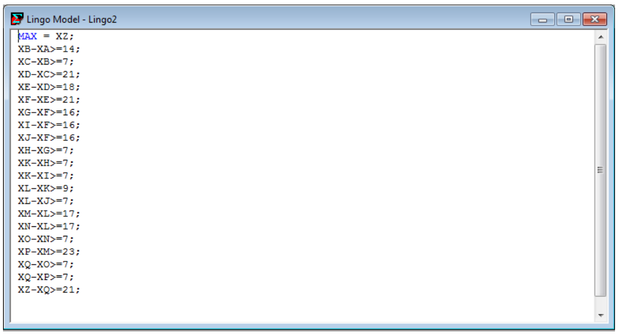

Figure 9. analysis for LP model using LINGO with aspiration level duration.

Solution of linear programming model through LINGO (Figure 9) result Project Duration of 188 days i.e. equation (13). Yielding f_0= 〚(C^T x)〛 _opt=188 days.

Therefore availability of resources, duration of some activity changes result final project duration reduces to 188 days from 196 days. Now introducing new variable λ that gives degree of satisfaction for optimization, $0 \leq \lambda \leq 1$ (represent as L in LINGO) the problem is reformulated using equation (14) where objective function has been converted to a constraints. Solution of Fuzzy linear programming model through LINGO (Figure 10) result minimum duration of 192 days with value of λ =0.5.

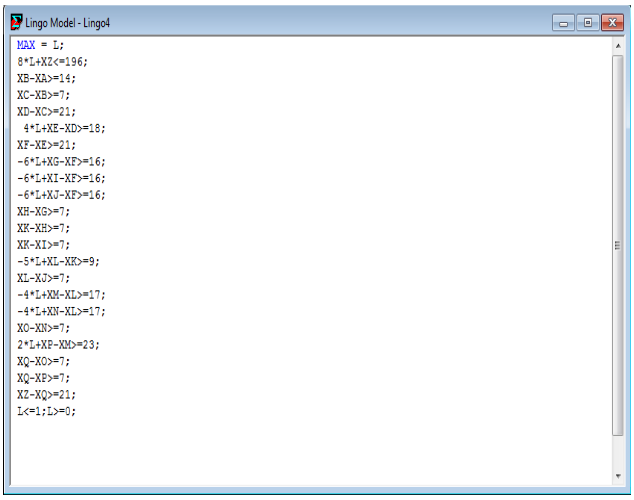

Figure 10. analysis for fuzzy LP model using LINGO.

IV. CONCLUSION

A construction project is considered successful if it is completed within stipulated time and budget with required quality and safety. So proper understanding and estimating of project duration is very important, crucial and sometime critical in controlling project or scheduling. Presence of uncertainties in various stages of activities may affect project completion duration that can incorporate into Fuzzy linear programming analysis. FLP allows flexibility is introduced in constraints, and objective function is transformed to another constraint. As In Case study reveal project duration reduce from 196 days to 192 days i.e. 2% flexibility in constraints, helping to identified optimum project duration in uncertain environment.

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