

Intercomparison of Rainfall Estimates of EV1 Distribution for Estimation of Peak Flood Discharge for Ungauged Catchments

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ABSTRACT

Estimation of Peak Flood Discharge (PFD) for a return period is one of the important parameters for planning, design and management of hydraulic structures such as dams, bridges, barrages and storm water drainage systems. For ungauged catchments, rainfall depth becomes an important input for estimation of PFD. The rainfall depth can be determined through Extreme Value Analysis (EVA), which involves fitting of probability distribution to the series of Annual 1-day Maximum Rainfall (AMR) data. In this paper, the AMR series derived from the daily rainfall data observed at Dehra site is used for EVA adopting Extreme Value Type-1 (EV1) distribution. Standard parameter estimation methods such as method of moments, method of least squares, maximum likelihood method, principle of maximum entropy, Probability Weighted Moments (PWM) and L-moments are applied for determination of parameters of the EV1 distribution. The adequacy of fitting of EV1 distribution adopted in EVA is evaluated by Goodness-of-Fit tests viz., Anderson-Darling and Kolmogorov-Smirnov (KS) and diagnostic tests viz., root mean squared error and mean absolute error. The KS and diagnostic tests results indicated that the PWM is better-suited method for determination of parameters of EV1 distribution, which is adopted for EVA of rainfall. The 1-hour distributed rainfall computed from the estimated extreme rainfall adopting EV1 (using PWM) distribution is used to estimate the PFD by rational formula. The estimated PFD for river Nakehr and its tributaries could be used for design of hydraulic structures.

Keywords: Anderson-Darling, Gumbel, Kolmogorov-Smirnov, Probability Weighted Moments, Rainfall, Peak Flood Discharge

I. INTRODUCTION

Estimation of Peak Flood Discharge (PFD) at a desired location on a river is important for planning, design and management of hydraulic structures such as dams, bridges, barrages and storm water drainage systems. These include different types of flood such as standard project flood, probable maximum flood and design basis flood. In case of large river basins, the hydrological and stream flow series of a significant duration are generally available. However, for ungauged catchments, stream flow data is not available other than rainfall. The rainfall data is also of shorter duration and may become an important input in derivation of PFD. For arriving at such design

values, Extreme Value Analysis (EVA) of rainfall is carried out.

Out of a number of probability distributions, Extreme Value Type-1 (EV1), commonly known as Gumbel, is generally adopted for EVA of rainfall (Casas, et al, 2011; Lee and Heo, 2011). EV1 distribution has no shape parameter as when compared to other distributions and this means that there is no change in the shape of Probability Density Function (PDF) (Singh et al., 2001). Moreover, EV1 distribution has the advantage of having only positive values, since the data series of rainfall are always positive (greater than zero). Lee et al. (2012) applied EV1 and Weibull distributions for estimation of extreme wind speed for

Korea region. They have found that EV1 distribution gives better results than Weibull. Daneshfaraz et al. (2013) carried out frequency analysis of wind speed adopting 2-parameter log-normal, truncated extreme value, truncated logistic and Weibull probability distributions and found that the truncated extreme value is the most appropriate distribution for Iran. Esteves (2013) applied EV1 distribution to estimate the extreme rainfall depths at different rain gauge stations in southeast United Kingdom. Rasel and Hossain (2015) adopted EV1 distribution for development of Intensity-Duration-Frequency (IDF) curves for seven divisions in Bangladesh. Likewise, Ewea et al. (2017) adopted EV1 distribution for development of IDF curves for the Kingdom of Saudia Arabia. Based on the review of research studies, it is noted that EV1 distribution is generally acceptable for EVA and hence adopted in the present study. Standard parameter estimation methods such as Method of Moments (MoM), Method of Least Squares (MLS), Maximum Likelihood Method (MLM), Principle of Maximum Entropy (PME), Probability Weighted Moments (PWM) and L-Moments (LMO) are applied for determination of parameters of the distribution (Arora and Singh, 1987). Number of studies has been carried out by different researchers on analyzing the characteristics of the parameter estimation methods of EV1 distribution.

Research reports indicated that MoM is a natural and relatively easy parameter estimation method (Landwehr et al., 1979). MLM and PME are considered the most efficient method, since it provides the smallest sampling variance of the estimated parameters and hence of the estimated quantiles compared to other methods. But, both MLM and PME have the disadvantage of frequently giving biased estimates and often failed to give the desired accuracy in estimating extremes from hydrological data (Ranyal and Salas, 1986). PWM and MLS are much less complicated, and the computations are simpler. Parameter estimates from small samples using PWM

and MLS are sometimes more accurate than the MLM estimates for EV1 distribution (Rasmussen and Gautam, 2003). LMO is linear combination of data, which is less influenced by outliers and the bias of their small sample estimates remains fairly small (Arora and Singh, 1987). But, there is no general agreement in applying particular method for estimation of rainfall for a region because of the characteristics of the estimators of EV1 distribution. Therefore, an attempt is made to apply six parameter estimation methods of EV1 distribution for EVA of rainfall. For quantitative assessment on rainfall data within the observed range, Goodness-of-Fit (GoF) tests viz., Anderson-Darling (AD) and Kolmogorov-Smirnov (KS) is applied. The 1-hour distributed rainfall derived from the estimated Extreme Rainfall (ER) adopting EV1 distribution and used to estimate the PFD for river Nakehr and its tributaries. The methodology adopted in EVA of rainfall, assessment of fitting of probability distribution using GoF and diagnostic tests, and estimation of PFD by rational formula are briefly described in the ensuing sections.

II. METHODS AND MATERIALS

The study is to estimate PFD at river Nakehr and its tributaries. Thus, it is required to process and validate the data for various application viz., (i) assess the adequacy of fitting of EV1 distribution to the AMR series using GoF tests; (ii) estimate the ER adopting EV1 distribution (using MoM, MLS, MLM, LMO and PWM); (iii) derive the 1-hour distributed rainfall from the estimated ER using CWC guidelines; (iv) compute the PFD using rational formula; and (v) analyze the results obtained thereof.

A. PDF and CDF of EV1 Distribution

The PDF and Cumulative Distribution Function (CDF) of the EV1 distribution are as follows:

$$\text{PDF: } f(r) = \frac{e^{-(r-\alpha)/\beta} e^{-e^{-(r-\alpha)/\beta}}}{\beta} \quad (1)$$

$$\text{CDF: } F(r) = e^{-e^{-(r-\alpha)/\beta}}, \beta > 0 \text{ and } (r = r_1, r_2, r_3, \dots, r_N) \quad (2)$$

where, α and β are the location and scale parameters of the distribution (Gumbel, 1960). The parameters are determined by MoM, MLS, MLM, PME, PWM and LMO, and used to estimate the ER (R_T) for different return periods from $R_T = \alpha + Y_T \beta$. Here, Y_T is called as reduced variate for a return period T (in year) and defined by $Y_T = -\ln(-\ln(1-(1/T)))$.

TABLE 1

VALUES OF COEFFICIENTS USED IN COMPUTATION OF SE

Parameter estimation method	A	B	C
MoM, PME and MLS	1.1589	0.1919	1.1000
PWM and LMO	1.1128	0.4574	0.8046
MLM	1.1087	0.5140	0.6079

The Standard Error (SE) on the estimated ER (Lieblein, 1974) is computed from Eq. (3) and given by:

$$SE(X_T) = (\beta/\sqrt{N}) (A + BY_T + CY_T^2)^{0.5} \quad (3)$$

where, r_i is the observed AMR of i^{th} observation. The values of the coefficients of A, B and C used in computation of SE by different methods of EV1 are presented in Table 1. The lower and upper confidence limits (LCL and UCL) of the estimated ER are obtained from the equations viz., $LCL = ER - 1.96(SE)$ and $UCL = ER + 1.96(SE)$.

B. Parameter Estimation Methods of EV1

The parameters of EV1 distribution are determined by six different methods, which are described below:

Method of Moments

$$\alpha = \bar{R} - (0.5772157)\beta \text{ and } \beta = (\sqrt{6}/\pi)S_R \quad (4)$$

where, \bar{R} and S_R are average and standard deviation of the observed AMR (AERB, 2008).

Method of Least Squares

$$\alpha = \bar{R} + \left(\left(\sum_{i=1}^N \ln(-\ln(P_i)) \right) \beta / N \right) \quad (5)$$

$$\beta = \frac{\left(\left(\sum_{i=1}^N r_i \right)^2 - \left(\sum_{i=1}^N r_i^2 \right) \right)}{\left(\left(\sum_{i=1}^N \ln(-\ln(P_i)) \right) - \left(\sum_{i=1}^N r_i \right) \left(\sum_{i=1}^N \ln(-\ln(P_i)) \right) \right)} \quad (6)$$

where, $P_i = (i - 0.44)/(N + 0.12)$. Here, $\ln(-\ln(P_i))$ defines the cumulative probability of non-exceedance for each r_i and N is the number of observations (Manik and Datta, 1998).

Maximum Likelihood Method

$$\alpha = -\beta \ln \left[\frac{N}{\sum_{i=1}^N \exp(-r_i/\beta)} \right] / N \quad (7)$$

$$\beta = \bar{R} - \left[\frac{\sum_{i=1}^N r_i \exp(-r_i/\beta)}{\sum_{i=1}^N \exp(-r_i/\beta)} \right] \quad (8)$$

Principle of Maximum Entropy

$$\alpha = \beta \ln \left(N / \sum_{i=1}^N \exp(-r_i/\beta) \right) \quad (9)$$

$$\beta = \bar{R} / \left(0.5772157 + \ln \left(N / \sum_{i=1}^N \exp(-r_i/\beta) \right) \right) \quad (10)$$

where, r_i is the observed data of i^{th} sample (Arora and Singh, 1987).

Probability Weighted Moments

$$\alpha = M_{100} - (0.5772157)\beta \text{ and } \beta = (M_{100} - 2M_{101})/\ln(2) \quad (11)$$

where,

$$M_{100} = (1/N) \sum_{i=1}^N r_i, \quad M_{101} = (1/N) \sum_{i=1}^N r_i (1 - F_i), \quad F_i = (i - 0.44)/(N + 0.12)$$

Here, 'i' is the rank assigned to each observation arranged in ascending order (Phien, 1987).

L-Moments

$$\alpha = b_0 - (0.5772157)\beta \text{ and } \beta = (2b_1 - b_0)/\ln(2) \quad (12)$$

where,

$$L_1 = b_0 = (1/N) \sum_{i=1}^N r_i, \quad L_2 = 2b_1 - b_0 \text{ and } b_1 = (1/(N(N-1))) \sum_{i=1}^N (i-1)r_i$$

Here, L_1 and L_2 are the first and second L-moments (Arora and Singh, 1987).

C. Goodness-of-Fit Tests

GoF tests viz., AD and KS statistic are applied for checking the adequacy of fitting of EV1 distribution (Zhang, 2002). Theoretical descriptions of GoF tests statistic are as follows:

$$AD = (-N) - (1/N) \sum_{i=1}^N \left\{ (2i-1) \ln(Z_i) + (2N+1-2i) \ln(1-Z_i) \right\} \quad (13)$$

$$KS = \max_{i=1}^N (F_e(r_i) - F_D(r_i)) \quad (14)$$

where, $Z_i = F(r_i) = (i - 0.44) / (N + 0.12)$ for $i=1,2,3,\dots,N$ with $r_1 < r_2 < \dots < r_N$, $F_e(r_i)$ is the empirical CDF of r_i , $F_D(r_i)$ is the derived CDF of r_i by EV1. For EV1 distribution, Gringorton plotting position formula is used for computation of derived CDF. The theoretical values AD and KS statistic for different sample size (N) at 5% significance level are available in the technical note on ‘Goodness-of-Fit Tests for Statistical Distributions’ by Charles Annis (2009).

Test criteria: If the computed values of GoF tests statistic given by different methods of EV1 distribution is less than that of theoretical values at the desired significance level (either at 5% or 1%), the method (s) would be taken as acceptable for EVA at that level.

D. Diagnostic Test

Sometimes the GoF test results would not offer a conclusive inference thus posing a problem for the user in selecting a suitable parameter estimation method of EV1 for their application. In such cases, a diagnostic test in adoption to GoF is applied for making inference. The selection of a suitable parameter estimation method of EV1 for EVA is performed through RMSE and MAE (Arora and Singh, 1987), which is defined as below:

$$RMSE = \left(\frac{1}{N} \sum_{i=1}^N (r_i - r_i^*)^2 \right)^{0.5} \quad (15)$$

$$MAE = \left(\frac{1}{N} \sum_{i=1}^N |r_i - r_i^*| \right) \quad (16)$$

where, r_i and r_i^* is the observed and estimated value of

AMR of i^{th} observation. The method has minimum RMSE and MAE is considered as best suited method of EV1 distribution for EVA.

III. APPLICATION

In this paper, efforts were made to estimate the PFD for different return periods for river Nakehr and its tributaries viz., Kher Nallah, Balha Nallah and Sour Nallah that contribute to flood flows, were carried out. The AMR series was extracted from the daily rainfall data observed at Dehra site during the period 1991 to 2017 and used for estimation of ER. Figure 1 presents the time series plot of the observed AMR. The descriptive statistics such as average, standard deviation, coefficient of variation, coefficient of skewness and coefficient of kurtosis of the observed AMR was determined as 76.2 mm, 28.7 mm, 37.7%, -0.308 and 0.655 respectively.

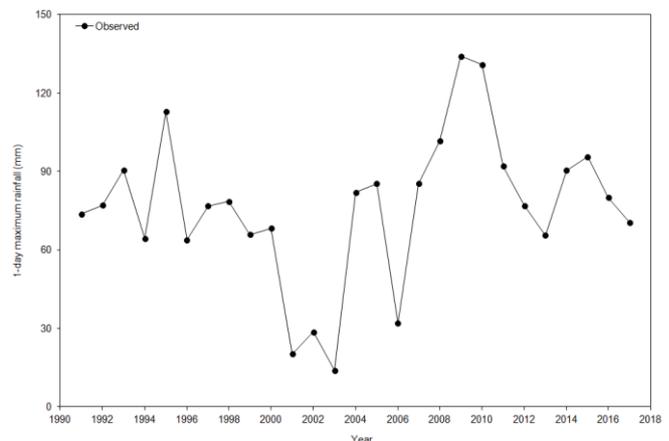


Figure 1. Time series plot of the observed AMR

IV. RESULTS AND DISCUSSIONS

A. EVA of Rainfall using EV1 Distribution

By applying the procedures of EV1 distribution, parameters were determined by six different methods and used for estimation of ER for different return periods. Table 2 gives the ER estimates with 95% confidence limits for different return periods adopting EV1 distribution. From Table 2, it is noted that the estimated ER obtained from MLM is comparative

higher than the corresponding values of other five methods. Also, from Table 2, it is noted that the SE on ER computed by PWM is less than the values obtained from other five methods. Figure 2 presents the plots of observed and estimated ER by different methods of EV1 distribution.

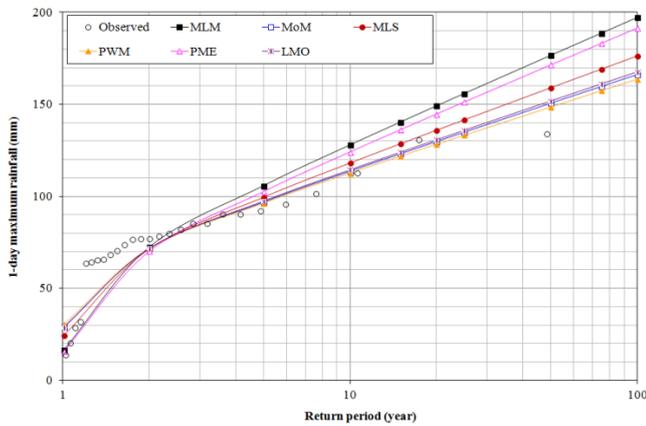


Figure 2. Plots of observed and estimated ER

B. Analysis Based on GoF Tests

The adequacy of fitting of EV1 distribution to the AMR series was performed by adopting AD and KS tests statistic, and GoF tests results are presented in Table 3.

TABLE 3

COMPUTED AND THEORETICAL VALUES OF GOF TESTS

GoF tests	Computed values					
	MoM	MLS	MLM	PME	PWM	LMO
AD	2.173	1.784	1.506	1.694	2.319	2.080
KS	0.208	0.222	0.226	0.253	0.202	0.204

From Table 3, it is noted that the computed values of AD by different parameter estimation methods of EV1 are greater than the theoretical value of 0.757 at 5% significance level, and at this level, the AD did not confirm the suitability of all six methods of EV1 distribution for EVA. In contrary to AD test, KS test results confirmed the applicability of all six methods of EV1 for EVA as the computed values are less than its theoretical value of 0.254 at 5% level.

C. Analysis Based on Diagnostic Test

As inferences drawn from GoF tests using AD and KS are contradictory to each other, the selection of an appropriate method for EVA was carried out by diagnostic test viz., RMSE and MAE. The diagnostic tests results are presented in Table 4. From diagnostic test results, it is noted that PWM method has minimum RMSE and MAE as when compared to other methods viz., MoM, MLS, MLM, PME and LMO and therefore ER obtained from PWM is considered for deriving of 1-hour distributed rainfall.

TABLE 4

DIAGNOSTIC TESTS RESULTS OF EV1 DISTRIBUTION

Diagnostic test	MoM	MLS	MLM	PME	PWM	LMO
RMSE	9.7	10.3	13.5	12.7	9.6	9.8
(Rank)	(2)	(4)	(6)	(5)	(1)	(3)
MAE	8.1	8.7	10.9	10.5	7.9	8.0
(Rank)	(3)	(4)	(6)	(5)	(1)	(2)
(Numbers given in brackets indicated the rank assigned to the method)						

The observed and estimated ER using EV1 (using PWM) is presented in Figure 3 along with confidence limits. From Figure 3, it can be seen that about 75% of the observed data are within the 95% confidence limits of the estimated ER obtained from EV1 (using PWM) distribution.

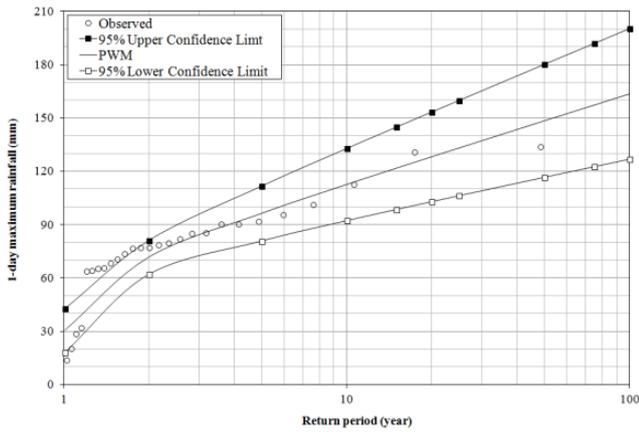


Figure 3: Plots of observed and estimated ER by EV1 (using PWM) distribution with confidence limits

D. Estimation of PFD

The estimated extreme rainfall was used to estimate the PFD for river Nakehr and its tributaries such as Kher Nallah, Balha Nallah and Sour Nallah that contribute to flood flows. The catchment areas of river Nakehr and its tributaries are presented in Table 5. From Table 5, it may be noted that the catchment areas vary from 2.24 km² and 142.30 km².

TABLE 2
ESTIMATED ER WITH LOWER AND UPPER CONFIDENCE LIMITS USING EV1 DISTRIBUTION

Return period (year)	MoM		MLS		MLM		PME		PWM		LMO	
	ER (mm)	SE (mm)										
2	71.5	5.0	71.4	5.6	72.5	6.7	70.2	6.5	71.6	4.9	71.9	5.1
5	96.8	8.5	99.5	9.5	105.9	10.2	102.7	10.9	96.2	7.9	97.6	8.3
10	113.6	11.5	118.1	12.8	128.1	13.1	124.2	14.8	112.6	10.4	114.6	10.9
15	123.1	13.3	128.6	14.7	140.6	14.8	136.3	17.0	121.8	11.9	124.2	12.4
20	129.7	14.5	135.9	16.1	149.3	16.1	144.8	18.7	128.2	12.9	130.9	13.5
25	134.8	15.5	141.6	17.2	156.1	17.0	151.4	19.9	133.2	13.7	136.1	14.3
50	150.5	18.6	159.0	20.6	176.9	20.0	171.5	23.8	148.4	16.3	152.0	17.0
75	159.7	20.4	169.2	22.6	188.9	21.7	183.3	26.1	157.3	17.8	161.3	18.5
100	166.1	21.7	176.3	24.0	197.5	23.0	191.6	27.8	163.6	18.8	167.8	19.6

TABLE 5
CATCHMENT AREA OF RIVER NAKEHR AND ITS TRIBUTARIES

S. No.	Name of catchment	Area (km ²)
1	Nakehr river (at RD 0)	142.30
2	Kher Nallah	32.97
3	Balha Nallah	7.15
4	Sour Nallah	2.24

Neither river Nakehr nor its tributaries are gauged and thus the PFD needed to be computed by indirect estimation from rainfall and catchment characteristics.

In general, for river Nakehr and its tributaries, the design storm duration is considered as 1-hour. From the estimated ER, the short duration rainfall (i.e., 1-hour) is obtained by using suitable conversion factor (Figure 4), as given in Central Water Commission Report titled ‘Flood estimation report for Western Himalayas-Zone 7’ (CWC, 1994) and presented in Table 6. This 1-hour rainfall was used as input (rainfall intensity) for computation of PFD of river Nakehr and its tributaries.

TABLE 6
DISTRIBUTED RAINFALL FOR SHORT DURATION

Return period (year)	Estimated extreme rainfall (mm)	1-hour distributed rainfall (mm)
2	71.6	30.4
5	96.2	40.9
10	112.6	47.8
15	121.8	51.7
20	128.2	54.5
25	133.2	56.6
50	148.4	63.1
75	157.3	66.9
100	163.6	69.5

Return period (year)	PFD (m ³ /s) for Nekahr river and its tributaries			
	Nekahr river (at RD 0)	Kher Nallah	Balha Nallah	Sour Nallah
2	142.3	33.0	7.2	2.2
5	281.2	65.2	14.1	4.4
10	662.2	153.4	33.3	10.4
15	890.0	206.2	44.7	14.0
20	1040.8	241.1	52.3	16.4
25	1125.9	260.9	56.6	17.7
50	1185.4	274.7	59.6	18.7
75	1231.3	285.3	61.9	19.4
100	1372.7	318.0	69.0	21.6

As mentioned these catchments are ungauged and hence the PFD for these catchments are computed by using rational formula, which is given below:

$$q = (0.278)CIA \tag{17}$$

where, q is peak discharge (m³/s), C is runoff coefficient, I is rainfall intensity (mm/hour) and A is catchment area (km²). By considering topography and general land use of the catchments, the value of the C is considered as 0.55. The computed PFD for different return periods for river Nakehr and its tributaries are presented in Table 7.

V. CONCLUSIONS

The paper described briefly the study carried out for EVA of rainfall for Dehra site adopting EV1 distribution (using MoM, MLS, MLM, PME, PWM and LMO) and estimation of PFD using CWC guidelines for river Nakehr and its tributaries such as Kher Nallah, Balha Nallah and Sour Nallah that contribute to flood flows. The conclusions drawn from the study were summarized, which are given below:

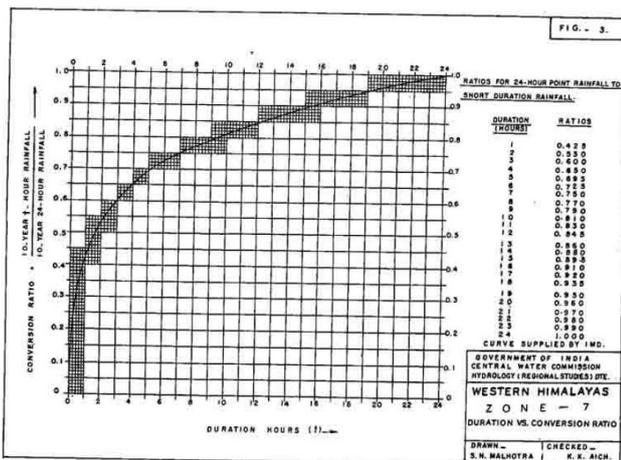


Figure 4. Conversion factor for computation of distributed rainfall for short duration (in hour)

TABLE 7
ESTIMATED PFD FOR RIVER NAKEHR AND ITS TRIBUTARIES

- i) KS test results confirmed the suitability of EV1 distribution (using MoM, MLS, MLM, PME, PWM and LMO) for EVA of rainfall. But, AD test results didn't support the use of EV1 distribution of EVA.
- ii) Diagnostic test results indicated that the PWM has minimum RMSE and MAE and therefore PWM could be considered as an appropriate method of EV1 for EVA of rainfall.
- iii) From Figure 2, it was observed that the about 75% of the observed data are within the confidence limits of the estimated ER by EV1 (using PWM).
- iv) The estimated ER by EV1 (using PWM) was used to compute 1-hour distributed rainfall adopting CWC guidelines, as described in Flood

estimation report for Western Himalayas-Zone 7.

- v) By using the 1-hour distributed rainfall, the PFD for river Nakehr and its tributaries was computed by rational formula. The study suggested that the PFD, as given in Table 7, could be considered for design of hydraulic structures.

However, considering the data length made available for the study, it was cautioned to use the PFD for return periods beyond 75-year because of uncertainty in the estimated values.

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